Number of Nonredundant Frequencies for Ground-Based Microwave Radiometric Temperature Profiling

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ABSTRACT

The information content of ground-based measurements of atmospheric microwave radiance in the 50–60 GHz band was studied, using a formulation due to Twomey. Weighting functions for ground-based microwave radiometry were constructed corresponding to measurements at various frequencies, and various elevation angles. Covariance matrices were computed for several sets of such weighting functions, and their eigenvalues were examined to estimate the degree of redundancy in the measurements.

1. Introduction

We are attempting to develop tropospheric temperature profilers for use in situations where logistical and other limitations make use of the radiosonde impractical. The ground-based 60 GHz microwave radiometer appears to be a compact, portable, near real-time profiler for such situations. A number of tradeoffs are involved in the selection of the operating characteristics of such an instrument, e.g., the number and frequency choice of channels, whether the instrument needs to scan in elevation angle, and others. Crucial to these choices is the question of the redundancy of the information provided by scanning capability or additional channels. To further our design studies we have applied a technique described by Twomey (1974, 1977) to the problem of temperature profiling with a ground-based 60 GHz microwave radiometer.

2. Theory

In the 20 to 60 GHz region, the measured brightness temperatures satisfy (to a good approximation) the following equation:

$$T_{b\nu} = \int_0^\infty T(s)\alpha_\nu(s) \exp\left[\int_0^s \alpha_\nu(s')ds'\right]ds + T_{b\nu}^\infty \int_0^\infty \exp[\alpha_\nu(s)ds] \quad (1)$$

where

$T_{b\nu}$	the downwelling microwave brightness temperature at frequency ν
$T(s)$	temperature at height s
$\alpha_\nu(s)$	the absorption coefficient
$T_{b\nu}^\infty$	the downwelling cosmic microwave background brightness temperature above the atmosphere.

Inferring atmospheric temperature structure from microwave brightness temperature measurements thus becomes the problem of inverting Eq. (1) for $T(s)$. Various techniques for performing this inversion are summarized in Westwater and Sweezy (1983).

Autocorrelation radiometry (Ruf 1987) provides a way to build microwave radiometers with a large number of channels and relatively good signal-to-noise figures. Consequently, it is useful to consider how much information is contained in such measurements, and how many frequencies can be usefully employed. The information content of indirect measurements has been discussed by Twomey and others (Twomey 1974, 1977, and references therein). Our analysis is based on the methods discussed by Twomey.

Equation (1) shows that a microwave brightness temperature measurement is, in effect, the measurement of a height weighted average of the atmospheric temperature over the vertical extent of the atmosphere in the radiometer antenna field of view. The weighting function $w_\nu(s)$, given by

$$w_\nu(s) = \alpha_\nu(s) \exp\left[\int_0^s \alpha_\nu(s')ds'\right] \quad (2)$$

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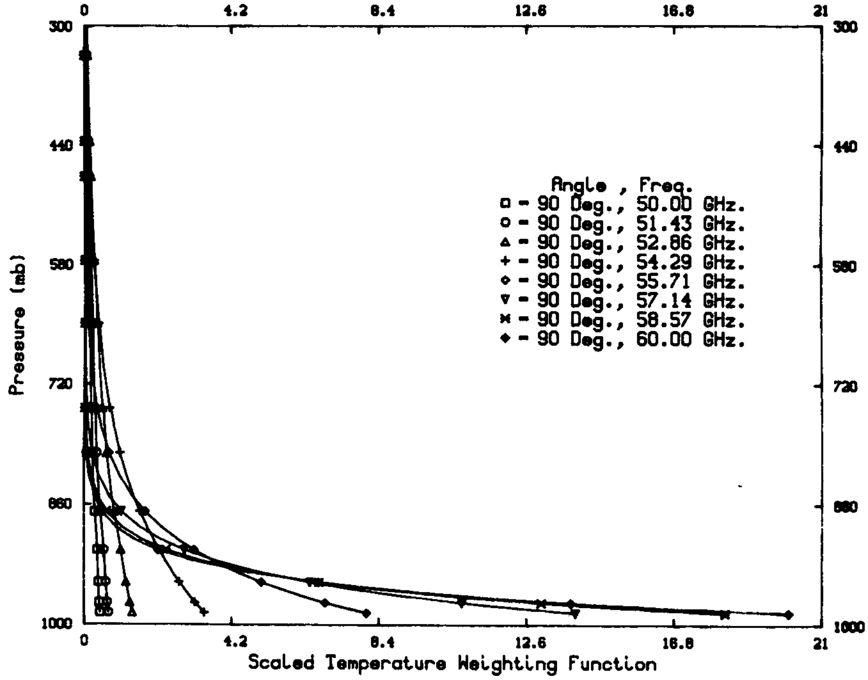

FIG. 1. Temperature weighting functions for various frequencies (arbitrarily scaled).

is only weakly dependent on the temperature, and hence can be considered to be (approximately) constant for a given altitude and frequency (for different profiles). (The approximate constancy of the $\alpha_\nu(s)$ dependent terms over various profiles holds only when absorption is strongly dominated by a well-mixed gas, as is the case with oxygen in the 50–60 GHz frequency region. For such a case, the main temperature effect is to cause density changes.) This splitting of the problem into profile dependent and (almost) profile independent parts permits a profile independent analysis of the measurement process. Figure 1 shows weighting func-

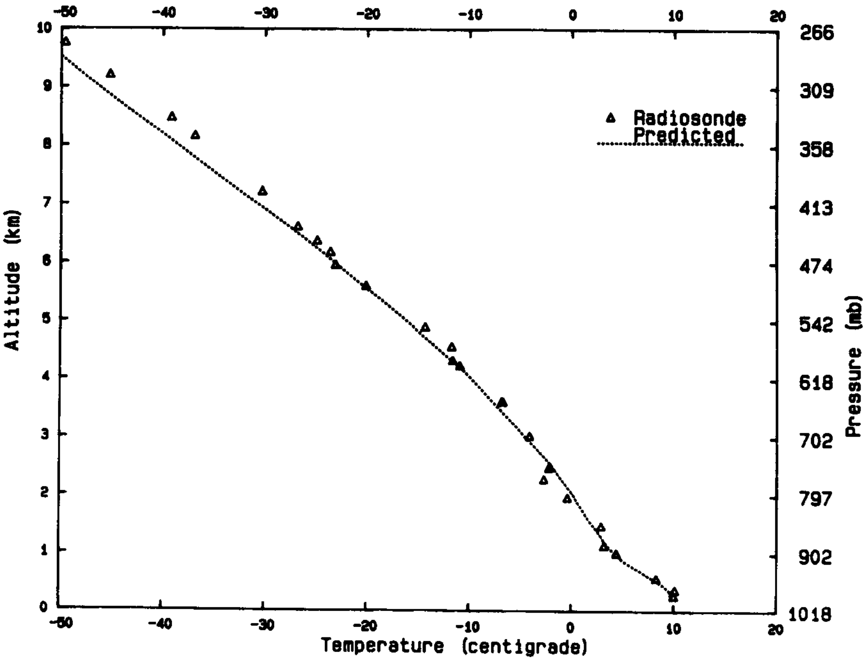

FIG. 2. The radiosonde temperature profile used as a basis for computing the weighting functions.

TABLE 1. Scaled covariance matrix for the weighting functions of Fig. 1 (eight frequencies, all zenith-pointing).

Frequency (GHz)	Frequency (6 Hz) 50.00	51.43	52.86	54.29	55.71	57.14	58.57	60.00
50.00	0.31	0.47	0.84	1.52	2.05	2.27	2.35	2.37
51.43	0.47	0.70	1.26	2.28	3.09	3.44	3.55	3.59
52.86	0.84	1.26	2.28	4.22	5.88	6.62	6.89	6.97
54.29	1.52	2.28	4.22	8.31	12.63	14.92	15.82	16.11
55.71	2.05	3.09	5.88	12.63	22.01	28.40	31.25	32.21
57.14	2.27	3.44	6.62	14.92	28.40	39.34	44.76	46.66
58.57	2.35	3.55	6.89	15.82	31.25	44.76	51.81	54.33
60.00	2.37	3.59	6.97	16.11	32.21	46.66	54.33	57.11

tions computed for several frequencies in the 50–60 GHz band. Strictly speaking, these weighting functions apply only to the particular cloud-free profile shown. As argued above, weighting functions are only weakly dependent on the details of the atmospheric profile as long as moisture is not considered. Our results only apply to clear air measurements.

Owing to the fact that for different frequencies the various levels of the atmosphere contribute differently to the measured brightness temperature, inference about the vertical structure of atmospheric temperature is possible. Qualitatively and intuitively we may say that the more similar the weighting functions, the less the information about atmospheric structure. Twomey shows that this notion may be made quantitative (Twomey 1977, pp. 185–212) by considering the eigenvalues of the covariance matrix of the weighting functions in relation to the accuracy of the measurements.

The elements C_{ij} of the covariance matrix **C** have the form

$$C_{ij} = \left| \int_0^\infty ds\, w_i(s) w_j(s) \right| . \tag{3}$$

Twomey (1974) shows that the number of redundant measurements is equal to the number of eigenvalues λ_j of **C** such that

$$\{\lambda_j / \sum_i \lambda_i\}^{1/2} < e \tag{4}$$

where e is the root-mean-square error in the brightness temperature measurement. [Actually, the corresponding expression occurring on page 944, paragraph 2 of Twomey's paper differs from the above, and is apparently a misprint, but Eq. (4) above follows from his reasoning.]

Ideally, the weighting functions would be independent of the parameters to be measured. Because the absorption coefficient depends on density, pressure, liquid water, water vapor, and temperature, this ideal is not achieved for our weighting functions. Consequently, we have computed weighting functions and covariance matrices for a variety of actual atmospheric radiosonde profiles. As expected, the differences in the weighting functions and the covariance matrix eigenvalues are generally small (see further comments in section 4). The results reported are based on the radiosonde observation shown in Fig. 2, which was selected as relatively featureless. Figure 2 also shows a temperature profile retrieved from computed brightness temperatures $T_{b\nu}$. The water vapor profile, not shown, was also used in computing the absorption coefficients, but its contribution is small at these frequencies.

TABLE 2. Eigenvalues and normalized relative eigenvalues for eight-frequency (50, 51.43, 52.86, 54.29, 55.71, 57.14, 58.57, 60 GHz), zenith-pointing (90 deg) measurements, for the covariance matrix of Table 1.

Eigenvalues	Square root relative eigenvalue
−.1709834E−06	.0000000E+00
.2135466E−04	.3426603E−03
.6558854E−04	.6005254E−03
.3755986E−02	.4544430E−02
.4233741E−01	.1525737E−01
.6245494E+00	.5860043E−01
.7334178E+01	.2008136E+00
.1738668E+03	.9777453E+00
.5559628E+03	.9312186E+00

TABLE 3. Eigenvalues and normalized relative eigenvalues for five-frequency (50, 52.25, 55, 57.50, 60 GHz), zenith-pointing (90 deg) measurements.

Eigenvalues	Square root relative eigenvalue
.2189952E−03	.1374410E−02
.4264755E−01	.1917989E−01
.2560232E+00	.4699360E−01
.4458683E+01	.1961112E+00
.1111741E+03	.9792662E+00

3. Results

A scaled covariance matrix **C** for the weighting functions of Fig. 1 is given in Table 1. Because of the scaling, only relative magnitudes of the terms have direct physical significance.

Measured antenna temperatures in the frequency range in question range from about 30 K to about 300 K. If we make the conservative performance assumption that the measurement is made with an accuracy of 1 kelvin, then e ranges from 0.003 to 0.03. If we adopt $e = 0.01$ as a compromise, then redundant measurements are expected to be present for $\{\lambda_j/\sum_i \lambda_i\}^{1/2} < 0.01$. Similarly, if the measurement can be made with an accuracy of 0.2 kelvin, redundancy sets in at a square-root-relative eigenvalue of 0.002. Tables 2–6 list eigenvalues λ_j and square roots of relative eigenvalues $\{\lambda_j/\sum_i \lambda_i\}^{1/2}$ for several frequency, elevation-angle measurement sets. Note that the individual eigenvalues are not to be associated with individual weighting functions, but are characteristic of the measurement set as a whole.

For the eight frequencies of Table 2 there are five or six significant measurements (depending on criterion above), with the rest clearly redundant. All five of the measurements of Table 3 appear to be significant for either criterion. Similarly, the three frequencies of Table 4 are all clearly significant.

If measurements are made at multiple elevation angles, a wider range of weighting functions becomes available. For three-frequency, three-elevation-angle measurements, five or six (depending on criterion) independent measurements are indicated (Table 5). With five frequencies and three elevation angles, the number of nonredundant measurements is either five or seven (Table 6).

4. Discussion and conclusions

Twomey (1974, 1977) emphasized that an analysis of the above type should be interpreted as an estimate, rather than an exact characterization of information content. We note that the above estimates of numbers of nonredundant measurements are likely to be overestimates, because several sources of error were unaccounted for. In particular, the variability of what we have called the weighting functions due to liquid water is unaccounted for. The contribution of the liquid water to the brightness temperature, which depends upon the amount of liquid water present, may be substantial for the lower frequencies of the range considered. Because the presence of liquid water changes the kernels, the analysis used here is not directly applicable to this case. For that reason we considered only the cloud-free case. It is possible to measure the amount of liquid water by other radiometric means (Hogg et al. 1983) and this information can be used to correct the oxygen channel measurements.

TABLE 4. Eigenvalues and normalized relative eigenvalues for three-frequency (50, 55, 60 GHz), zenith-pointing (90 deg) measurements.

Eigenvalues	Square root relative eigenvalue
.4921771E−01	.2622579E−01
.3858543E+01	.2322093E+00
.6765124E+02	.9723122E+00

TABLE 5. Eigenvalues and normalized relative eigenvalues for three-frequency (50, 55, 60 GHz), three-elevation-angle (90, 30, 9 deg) measurements.

Eigenvalues	Square root relative eigenvalue
.7696883E−06	.3207428E−04
.1708489E−03	.4778655E−03
.9290342E−03	.1114334E−02
.1472228E−01	.4435954E−02
.1396741E+00	.1366336E−01
.1178989E+01	.3969670E−01
.1163352E+02	.1246968E+00
.8666693E+02	.3403503E+00
.6485363E+03	.9310364E+00

These results show that five to seven independent measurements are the most that can be expected in the 50–60 GHz frequency band under the conditions of this study. The results also seem to suggest that more nonredundant measurements are available if a significant improvement in relative error of measurement can be achieved, e.g., sufficient for changing the marginal relative eigenvalues to acceptable. This conclusion may not be justified unless methods for dealing with the sources of error discussed in the previous paragraph can be found.

TABLE 6. Eigenvalues and normalized relative eigenvalues for five-frequency (50, 52.25, 55, 57.50, 60 GHz), three-elevation-angle (90, 30, 9 deg) measurements.

Eigenvalues	Square root relative eigenvalue
−.4727811E−06	.0000000E+00
.1480877E−06	.1109172E−04
.2383822E−05	.4450165E−04
.9155900E−05	.8721471E−04
.9760167E−05	.9004671E−04
.9092617E−04	.2748424E−03
.1922540E−03	.3996473E−03
.1027226E−02	.9237879E−03
.9476373E−02	.2805823E−02
.1117245E+00	.9634150E−02
.4865010E+00	.2010394E−01
.2410316E+01	.4474827E−01
.1770869E+02	.1212921E+00
.1310553E+03	.3299640E+00
.1051926E+04	.9348281E+00

These methods and results are important for deciding how many frequencies an operational profiler should measure and whether it needs an elevation angle scanning capability. The methods can also contribute to finding the best choice of frequencies at which to operate, since the information content depends not only on the number of frequencies measured, but also on the particular choice of frequencies made. They cannot fully answer the even more important question of the accuracy of the retrieved temperature profile, because that depends on more than just the independence of the kernels. In particular, our analysis does not treat the error due to the uncertainty in the kernels.

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