

A Comparison of Several Radiometric Methods of Deducing Path-Integrated Cloud Liquid Water

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ABSTRACT

Using radiometer data collected during the Canadian Atlantic Storms Program, we have investigated five different methods of estimating the path-integrated, or columnar, cloud liquid water. The methods consist of one- and two-channel physical retrievals, the standard method of linear statistical inversion using two channels, and two statistical methods that proceed from an initial determination of several empirical regressions between measured and computed quantities. Though differing in details and complexity, the methods gave estimates of cloud liquid that did not deviate greatly from one another. We assessed the accuracy of the methods by simulation. Using hypothetical profiles of cloud liquid in archival soundings, we calculated the atmospheric emission and thus the brightness temperatures that would be measured in the two channels of the radiometer. These values were taken as data for the five methods, and the amount of liquid was calculated. Results showed that the three statistical methods were more accurate than the physical methods, but no one of the three was significantly better than the others. In the four methods requiring measurements in two channels, the columnar water vapor is computed as part of the retrieval procedure. A comparison of the computed with the actual vapor amounts showed that one of the statistical methods employing empirical regressions was the most accurate for vapor retrieval. For this optimum method, the rms deviation of the measured columnar liquid from its actual value was 0.159 mm and the rms deviation of the columnar vapor was 0.867 mm. As fractions of the overall average liquid and vapor in the simulations, these deviations amount to 37% and 8.7% respectively. If cases are excluded in which the liquid amount is small or nonexistent, the fractional deviation of the liquid estimates decreases and that of the vapor increases.

1. Introduction

The use of ground-based radiometers for measuring the tropospheric temperature profile and the vertically integrated amounts of water vapor and cloud liquid is well established. A recent review of the subject, which includes a description of the different uses of the data, is that of Askne and Westwater (1986). Comparisons of radiometer measurements with radiosonde data have shown that useful accuracy can routinely be achieved in temperature profiling and in the measurement of columnar water vapor (Decker et al. 1978; Hogg et al. 1983a; Blaskovic et al. 1989). In each of the comparisons cited, the instrument used was a six-channel radiometer, relying on two channels for the liquid and vapor measurements and four additional channels to permit temperature profiling. For the measurement of vapor and liquid only, simpler dual-channel equipment

is used (Guiraud et al. 1979; Hogg et al. 1983b; Bhattacharya and Uppal 1987; Wei et al. 1987; Heggli and Rauber 1988). The principles of dual-frequency measurement of liquid and vapor, first proposed by Staelin (1966), were laid out by Westwater (1978). Most current research relies on either the physical method of estimating the liquid and vapor, as explained in the Westwater (1978) analysis, or the method of linear statistical inversion as given by Westwater and Guiraud (1980).

Evaluations of the accuracy of radiometric measurements are usually based on comparisons with radiosonde observations. For the water vapor this comparison is not ideal, because 1) the measurement of humidity by the radiosonde is inexact and 2) the humidity varies with time and position and so may have a different integrated value along the instantaneous, straight-line path of the radiometer than over the irregular course followed by the ascending balloon. Nevertheless, the measurements are sufficiently comparable that the comparisons have meaning. The good agreement that is usually found provides strong support for radiometric measurements of columnar vapor. For liquid, comparisons with independent observations are nearly impossible because the radiosonde does not measure liquid water. Even if it did, comparisons would

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be unconvincing. Cloud liquid has a higher degree of variability than vapor, which would make comparisons inappropriate unless the measurements were taken very nearly simultaneously and along coincident paths.

We are only aware of one experimental assessment of the accuracy of path-integrated liquid measured by radiometer, namely, that reported by Snider et al. (1980). The radiometer was a dual-frequency instrument operating at 20.6 GHz (sensitive mainly to the water vapor emission) and 31.65 GHz, sensitive mainly to the liquid. Its antenna was pointed in the direction of the COMSTAR-3 geosynchronous satellite. The method of linear statistical inversion was used to derive the integrated liquid and vapor along this path. An independent estimate of the path-integrated liquid was obtained by simultaneously measuring the attenuation of a 28-GHz signal transmitted by the satellite and the brightness temperature at the same frequency. Because both the attenuation and the emission of the liquid were measured in this second method, Snider et al. regarded it as the more accurate of the two means of estimating the liquid along the path. Both instruments operated at Denver, Colorado for several months. Generally good agreement was found between the liquid amounts determined by the two methods. A regression analysis based on approximately 18 hours of data during cloudy conditions on 8 days in August 1979, showed a correlation coefficient of 0.93 between the two datasets and an rms difference of 0.29 mm. The average liquid amount during this time was 0.58 mm and the highest value was 5.2 mm. On the average, the liquid amount estimated from the dual-frequency measurement exceeded that of the more accurate system by 11%. The authors regarded the agreement as good, but explained that there were several possible explanations for the discrepancies. Although the measurements were simultaneous and along the same path, the beam widths of the systems were not the same. Also, the satellite-monitoring system did not have the same effective beam width for its emission measurement as for its attenuation measurement. Finally, the two instruments had different effective sampling times. The implication was that the agreement was good, but it might have been even better if the measurements had been more nearly comparable.

McGill University collaborated with the Atmospheric Environment Service of Canada in operating a six-channel radiometer during January–March 1986 as part of the Canadian Atlantic Storms Program (CASP). The equipment was located at the project headquarters in Shearwater, Nova Scotia. An earlier report (Blaskovic et al. 1989) described the equipment and presented an analysis of the accuracy of the measurements of temperature and the vertically integrated water vapor, based on a comparison of these measurements with data from 108 special radiosondes taken at the radiometer site. Liquid water was also measured during this project, but the observations were not in-

cluded in the earlier report because we had no way to assess the accuracy of the measurements.

In the present report, we describe five different methods of estimating the amount of path-integrated cloud liquid from radiometer measurements and apply these methods to portions of the data collected during CASP. We assess the accuracy of the methods by simulation, comparing the amount of liquid inferred to be present with the amount prescribed in a cloud model. We show that a new method, based on empirical regressions, is capable of measuring the liquid with about the same accuracy as the method of linear statistical inversion. The new method has the advantage of providing a slight improvement in the accuracy of vapor estimates in the presence of large amounts of liquid.

2. Fundamentals

Although the methods of microwave radiometry are becoming known to an increasingly wide community of researchers, the concepts may be unfamiliar to some readers. We therefore begin by outlining the fundamentals and introducing the terminology and notation to be used in the paper.

The monochromatic radiance incident at the earth's surface ($z = 0$) from the zenith direction may be written

$$I(0) = I_e t(0, \infty) + \int_0^\infty B(z) \gamma(z) t(0, z) dz, \quad (1)$$

where I_e is the extraterrestrial radiance, B is the Planck function, and γ is the volume absorption coefficient of the atmosphere. The transmittance t is defined by

$$t(0, z) = \exp[-\tau(0, z)], \quad (2)$$

where τ is the optical thickness, given by

$$\tau(0, z) = \int_0^z \gamma(y) dy. \quad (3)$$

All the quantities in (1) are spectral quantities, having values that depend on the wavelength of the radiation. This equation includes the effects on radiative transfer of absorption and emission but not scattering. It is a good approximation at microwave frequencies for a clear or cloudy atmosphere, but may not be satisfactory for an atmosphere with drizzle drops, raindrops, or wet snowflakes, for which the effects of scattering will not be negligible compared with absorption. Restricting our attention to conditions without precipitation, we may write for the absorption coefficient

$$\gamma = \gamma_a + \gamma_l, \quad (4)$$

where γ_a is the absorption coefficient of clear air and γ_l is that of cloud. The subscript l is used for cloud, because only the liquid phase of condensed water interacts significantly with microwaves.

For atmospheric emission in the microwave region of the electromagnetic spectrum, the Rayleigh–Jeans

approximation may be used for the Planck function, and (1) becomes

$$T_b = T_{bet}(0, \infty) + \int_0^\infty T(z)\gamma(z)t(0, z)dz. \quad (5)$$

Here $T(z)$ is the temperature profile, T_b is the brightness temperature at the surface, which is defined as the equivalent black body temperature corresponding to the radiance $I(0)$, and T_{be} is the brightness temperature of the extraterrestrial radiance. A microwave radiometer is calibrated to measure T_b .

From (2) and (3), it follows that

$$\frac{\partial t}{\partial z} = -t(0, z)\gamma(z),$$

so that (5) may be written

$$T_b = T_{bet}(0, \infty) + \int_0^\infty T(z)\left(-\frac{\partial t}{\partial z}\right)dz. \quad (6)$$

This equation may be integrated by introducing the mean radiating temperature of the atmosphere, T_m , such that

$$T_b = T_{bet}(0, \infty) + T_m \int_0^\infty \left(-\frac{\partial t}{\partial z}\right)dz.$$

Thus

$$T_b = T_{bet}(0, \infty) + T_m[1 - t(0, \infty)],$$

from which

$$t(0, \infty) = \left(\frac{T_m - T_b}{T_m - T_{be}}\right) \quad (7)$$

and

$$\tau(0, \infty) = -\ln\left(\frac{T_m - T_b}{T_m - T_{be}}\right). \quad (8)$$

The total atmospheric optical thickness may be determined from (8), given a measurement of T_b and estimates of T_m and T_{be} .

From (4), the optical thickness may be written

$$\tau(0, \infty) = \tau_a + \tau_l, \quad (9)$$

where τ_a is the optical thickness of the clear air and τ_l is that of the cloud. The only significant clear air absorbers in the microwave spectrum are oxygen and water vapor. The total atmospheric optical thickness at a given microwave frequency may therefore be expressed as the sum of the optical thicknesses of water vapor, oxygen, and cloud liquid water:

$$\tau = \tau_V + \tau_O + \tau_l. \quad (10)$$

The optical thickness of the vapor is related to the vapor density ρ_V by

$$\tau_V = \int_0^\infty \rho_V k_V dz = K_V V, \quad (11)$$

where k_V is the mass absorption coefficient, $V = \int \rho_V dz$ is the total columnar vapor (or "precipitable water"), and K_V is a weighted mean value of k_V ; V has SI units of kg m^{-2} , but is conventionally measured in cgs units of g cm^{-2} or, more commonly still, in terms of the volume equivalent, which is $\text{cm}^3 \text{cm}^{-2}$ or cm ; k_V and K_V have units of $\text{m}^2 \text{kg}^{-1}$ or, more commonly, $\text{cm}^2 \text{cm}^{-3} = \text{cm}^{-1}$. Similarly, the optical thickness of the cloud liquid is related to the liquid water content M (the mass of water per unit volume of air) by

$$\tau_l = \int_0^\infty M k_l dz = K_l L, \quad (12)$$

where k_l is the mass absorption coefficient of the cloud water, K_l is its weighted average, and L is the total columnar liquid water. Units are the same as for the vapor. For oxygen the optical thickness is

$$\tau_O = \int_0^\infty \rho_O k_O dz, \quad (13)$$

where $\rho_O = m_O \rho$ is the density of oxygen, with ρ the air density and m_O the (constant) mixing ratio of oxygen, and k_O is the mass absorption coefficient of oxygen.

The relationships given in this section provide the background for the different methods of determining liquid water that follow.

3. Physical methods

a. One-channel physical method (Method 1)

The simplest approach to determining the columnar liquid by radiometry follows from (10) and (12),

$$L = (\tau - \tau_V - \tau_O)/K_l, \quad (14)$$

where τ is determined through (8) by a measurement of the brightness temperature, and τ_V , τ_O , and K_l are assumed constant and estimated from calculations with average soundings or model atmospheres. The frequency chosen for such a measurement should be one for which the liquid absorption is large compared with that of vapor and oxygen, so that $\tau > (\tau_V + \tau_O)$ in (14). In our experiments, a frequency of 31.65 GHz was employed, which lies between the water vapor absorption line at 22.2 GHz and the peak in the oxygen band at 60 GHz.

b. Two-channel physical method (Method 2)

A shortcoming of the one-channel method is that τ_V is not constant, but varies with the amount of vapor present. An improvement in the accuracy of estimating L can be achieved by measuring the brightness temperatures at two frequencies, one sensitive to vapor emission, the other to liquid. Denoting these frequencies by subscripts 1 and 2, we may write

$$\left. \begin{aligned} \tau_1 &= K_{V1}V + K_{I1}L + \tau_{O1} \\ \tau_2 &= K_{V2}V + K_{I2}L + \tau_{O2} \end{aligned} \right\}, \quad (15)$$

from which we can solve for L and V :

$$\left. \begin{aligned} L &= \frac{f_2 K_{V1} - f_1 K_{V2}}{K_{V1} K_{I2} - K_{V2} K_{I1}} \\ V &= \frac{f_1 K_{I2} - f_2 K_{I1}}{K_{V1} K_{I2} - K_{V2} K_{I1}} \end{aligned} \right\}, \quad (16)$$

where $f_1 = (\tau_1 - \tau_{O1})$ and $f_2 = (\tau_2 - \tau_{O2})$. Here τ_1 and τ_2 are determined from measured brightness temperatures, and the quantities τ_O , K_V , and K_I , for both frequencies, are constants determined by calculation from a mean sounding or a model atmosphere. First proposed by Staelin (1966), this dual-channel method is the basis of most current research. The frequencies used in our experiments are 20.6 GHz, sensitive mainly to water vapor, and 31.65 GHz, sensitive to liquid water. These frequencies were introduced by researchers at the Wave Propagation Laboratory (Westwater 1978; Guiraud et al. 1979) and have tended to become the standard for dual-channel radiometry. For convenience in comparing this method with some of the others, (16) can be written as

$$\left. \begin{aligned} L &= A_0 + A_1 \tau_1 + A_2 \tau_2 \\ V &= B_0 + B_1 \tau_1 + B_2 \tau_2 \end{aligned} \right\}, \quad (17)$$

where

$$\begin{aligned} A_0 &= (\tau_{O1} K_{V2} - \tau_{O2} K_{V1}) / F \\ A_1 &= -K_{V2} / F \\ A_2 &= K_{V1} / F \\ B_0 &= (\tau_{O2} K_{I1} - \tau_{O1} K_{I2}) / F \\ B_1 &= K_{I2} / F \\ B_2 &= -K_{I1} / F \end{aligned}$$

and

$$F = K_{V1} K_{I2} - K_{V2} K_{I1}.$$

4. Statistical methods

a. Linear statistical inversion (Method 3)

Although the dual-channel method is an improvement over the single-channel approach, the estimates of L and V are still subject to uncertainty because of the variability of the parameters τ_O , K_V , and K_I about their average or climatological values. Westwater and Guiraud (1980) showed that this variability is most significant in K_I , and that the ratio of the standard deviation to the mean value of this quantity can amount to 25%. They explained that a modest improvement over Method 2 can be achieved by a more complex and less straightforward method of handling

the dual-channel measurements, namely, linear statistical inversion.

We denote by \mathbf{d} the data vector, consisting of pairs of values (τ_1, τ_2) determined from (8) by the brightness temperatures of the two channels. From these measurements, we wish to determine the vector \mathbf{p} , consisting of pairs of values (L, V) . Linear statistical inversion provides an estimate $\hat{\mathbf{p}}$ of the vector \mathbf{p} by the relation

$$\hat{\mathbf{p}} = \langle \mathbf{p} \rangle + \langle \mathbf{p}'\mathbf{d}'^T \rangle \langle \mathbf{d}'\mathbf{d}'^T \rangle^{-1} \mathbf{d}',$$

where $\langle \rangle$ denotes an ensemble average, a prime denotes the deviation from the average (thus $\mathbf{d}' = \mathbf{d} - \langle \mathbf{d} \rangle$), and \mathbf{d}'^T denotes the transpose of vector \mathbf{d} . The ensemble averages are determined from a representative set of radiosonde profiles, in some of which clouds with prescribed water content are assumed to be present. From these are calculated L and V , making up the vector \mathbf{p} , and also the brightness temperatures for both channels, which can be converted through (8) to τ_1 and τ_2 , and thus the vectors \mathbf{d} needed for the ensemble averages. This procedure leads to equations for columnar liquid and vapor of the form

$$\left. \begin{aligned} L &= a_0 + a_1 \tau_1 + a_2 \tau_2 \\ V &= b_0 + b_1 \tau_1 + b_2 \tau_2 \end{aligned} \right\}, \quad (18)$$

where the a_i and b_i are called the statistical inversion coefficients.

b. Method of empirical regressions (Method 4)

Chinese researchers (Huang and Zou 1987; Wei et al. 1987) have demonstrated the application of a method based on empirical regressions to determine L and V from dual-channel measurements. From a representative set of soundings, some of which contain cloud with prescribed water content, values are calculated for the quantities L , V , T_{b1} , T_{b2} , τ_1 , τ_2 , τ_{a1} , and τ_{a2} , where τ_a as in (9) denotes the optical thickness of the clear air and τ as in (10) denotes the total optical thickness. Subscript 1 refers to the low (vapor sensitive) frequency and subscript 2, the high (liquid sensitive) frequency.

Three empirical regressions are determined, denoted by A, B, and C. Regression A is a polynomial fit to a plot of L versus T_{b2} , which we can indicate as

$$\text{A: } L = L(T_{b2}). \quad (19)$$

Regression B is a linear fit to a plot of τ_2 versus L , of the form

$$\text{B: } \tau_2 = \tau_{a2} + \tau_{l2} = \tau_{a2} + qL. \quad (20)$$

Regression C is a linear fit to a plot of V versus τ_{a1} , denoted as

$$\text{C: } V = m + n\tau_{a1}. \quad (21)$$

Given these regressions, the procedure of determining

L and V from measurements of T_{b1} and T_{b2} is as follows:

- 1) Using A, L is determined directly from T_{b2} .
- 2) Using B, τ_{l2} is determined from L .
- 3) Using (8), τ_1 is calculated from T_{b1} .
- 4) τ_{l1} is calculated from τ_{l2} using the proportionality

$$\tau_{l1} = r\tau_{l2}.$$

In the work cited, the proportionality factor r was expressed as an analytical function of the frequency ratio. In this paper, it was determined empirically, as will be explained later.

- 5) τ_{a1} is calculated from $\tau_{a1} = \tau_1 - \tau_{l1}$.
- 6) Using C, V is determined from τ_{a1} .

Much of the detail of Method 4 is thus devoted to determining V . These steps are summarized in the following expression for V :

$$V = b_0 + b_1\tau_1 + b_3L, \quad (22)$$

where $b_0 = m$, $b_1 = n$, and $b_3 = -nrq$.

c. Method of empirical regressions with iteration (Method 5)

Method 5 is an extension of Method 4 requiring an additional regression relation determined from a set of representative soundings. Denoted by D, this is a linear fit to a plot of τ_{a2} versus V , denoted by

$$D: \tau_{a2} = x + yV. \quad (23)$$

Given this regression along with the results from Method 4, Method 5 proceeds as follows:

- 1) Calculate τ_2 from T_{b2} using (8).
- 2) Using the value of V determined in Method 4, calculate τ_{a2} from (23).
- 3) Determine τ_{l2} from $\tau_{l2} = \tau_2 - \tau_{a2}$.
- 4) Calculate a new value of L from (20).
- 5) Use this value of L in (22) to calculate a new value of V .
- 6) Recalculate L using (20) and iterate.

The equation for L in step (4) is seen to be

$$L = \frac{1}{q} [\tau_2 - (x + yV)]. \quad (24)$$

Comparing this with (22), it is clear that the iterative solution for L converges to the value

$$L = \frac{1}{q(1 - ynr)} [-(x + ym) - ynr\tau_1 + \tau_2], \quad (25)$$

and that the columnar vapor is given by

$$V = \frac{1}{(1 - ynr)} [(m + nxr) + n\tau_1 - nr\tau_2]. \quad (26)$$

These expressions are thus linear in τ_1 and τ_2 , the same in form as (16) and (18), but with different coefficients.

5. Implementation of the methods

We used the radiometer data collected at Shearwater, Nova Scotia, during the Canadian Atlantic Storms Program to calculate the columnar liquid by each of the methods described in the preceding section. The instrument was a six-channel radiometer built for the Atmospheric Environment Service of Canada on a design very similar to that of the Wave Propagation Laboratory. The frequencies employed are 20.6 and 31.65 GHz for vapor and liquid determination, and four additional frequencies near the 60 GHz peak in the oxygen spectrum to enable the estimation of the temperature profile. The instrument, the calibration procedure, the data, and the accuracy of the temperature profile estimates were described by Blaskovic et al. (1989). That paper also examined the accuracy of columnar vapor estimates using what we now call Method 3, which at the time of CASP was the only method we were equipped to use. Estimates of columnar liquid were produced during CASP as part of the standard analysis, but we did not discuss them in the earlier paper because of doubts concerning the method and the lack of any independent data on liquid water for comparison; however, the full set of liquid estimates based on Method 3 for the 52 days of radiometer operation are included in a report summarizing the radiometer observations (Wei and Rogers 1988).

All methods of liquid estimation begin as a first step with the conversion of a measured brightness temperature to the corresponding optical thickness through (8). This requires knowledge of T_{be} and T_m . For the extraterrestrial radiance or cosmic background, we take $T_{be} = 2.9$ K (Hogg et al. 1983a), whose value is so small compared with T_m as to be negligible for most purposes. An indirect means is used for estimating T_m . In preparing the radiometer and its data system for CASP, B. E. Sheppard of the AES used five years of twice-daily radiosonde records from Shelburne, Nova Scotia, to derive the statistical inversion coefficients for Method 3. After eliminating the soundings in which the data were incomplete or questionable, he computed for each of the remaining soundings the total optical thickness at each of the radiometer frequencies and, using (5), the corresponding brightness temperatures. Every sounding thus provides a pair of values of T_b and $\tau(0, \infty)$ which, through (8), can be used to calculate T_m . Sheppard found that T_m , calculated this way, was closely correlated with the surface temperature, T_s . Regression analysis showed that for each of the six frequencies and to good approximation,

$$T_m = T_0 + (T_s - 273.15)\mu. \quad (27)$$

For 20.6 GHz, $T_0 = 264.38$ K and $\mu = 0.8788$; for 31.65 GHz the corresponding values are 263.36 K and 0.8814. The rms deviations from these regressions are 3.5 K and 3.9 K, respectively, for the two frequencies. In applying (8) to convert from T_b to $\tau(0, \infty)$, T_m is thus estimated from a measurement of the surface

TABLE 1. Retrieval coefficients for Method 1.

	τ_V	τ_O	K_l (cm^{-1})
Mean	0.01405	0.02442	1.82173
Standard deviation	0.0065	0.0016	0.6876

temperature. The uncertainty in T_m leads to an uncertainty in the optical thickness. It may be shown from (8) that, for typical values of T_b and T_m , the fractional error in τ is approximately equal to the fractional error in T_m . From the results of Sheppard's regression, we may thus infer that the rms fractional error in τ , arising from uncertainties in T_m , is approximately $4/260 = 1.5\%$.

We have just seen that the estimation of the mean radiating temperature depends on the coefficients T_0 and μ that are determined from a climatologically representative set of atmospheric soundings. All the methods of radiometric estimation of liquid depend on these and other constants or "retrieval coefficients" that are derived from radiosonde data. Given the temperature, pressure, and relative humidity as functions of height, we must be able to calculate the brightness temperature and the optical thicknesses of the clear air and of any cloud that may be present to derive the coefficients. This procedure requires knowledge of the absorbing properties of oxygen, water vapor, and cloud, details of which will now be given.

a. Clear-air absorption coefficients

For the absorption coefficient of oxygen we used the approximation of Rosenkranz (1975),

$$\gamma_O = Cp(\nu/T)^2 F,$$

where p is the pressure, T the temperature, and ν the frequency. When temperature is given in K, pressure in mb, and frequency in GHz, γ_O is in km^{-1} for C equal to 0.330. The function F , given by Rosenkranz as a series, depends on frequency and includes the effects of line shape, pressure broadening, and the interference between the lines in the spectrum. This formulation is widely used in ground-based microwave radiometry.

Water vapor absorption is approximated using the formulation of Hogg et al. (1983c), which includes the effect of continuum absorption and the resonance line at 22.235 GHz in the water vapor spectrum. Too complicated to repeat here, the formulation enables us to

compute γ_V , given the values of radio frequency, temperature, pressure, and vapor density ρ_V .

b. Cloud absorption coefficient

Absorption by liquid water droplets depends on temperature and increases monotonically with frequency in the microwave range. We used the formulation of Westwater (1972) for the absorption coefficient. It assumes that the diameter of the drops is much smaller than the wavelength and is therefore appropriate for cloud droplets in the microwave spectrum but not for raindrops. The formulation allows the computation of γ_l , given the frequency, temperature, and cloud liquid water content M .

c. The cloud model

To include the effects of clouds in radiative transfer calculations we must specify the water content M as a function of altitude. Although radiosonde data provide an approximate indication of the presence or absence of cloud through the relative humidity, the water content is not measured by radiosonde and such information is unavailable from any significant archival records. It is therefore necessary to employ a hypothetical cloud water profile or cloud model to carry out calculations that include the radiative effects of liquid.

We use the cloud model that Sheppard (1981) employed when he calculated the retrieval coefficients for Method 3. It is assumed that cloud is present for any interval of altitude in which the relative humidity exceeds 95%. At each level h within the cloud, a hypothetical liquid water content M^* , called the adiabatic liquid water content, is then calculated from the formula

$$M^*(h) = \int_{z_0}^h \rho(c_p/\mathcal{L})(\Gamma - \Gamma_s) dz, \quad (28)$$

where z_0 is the altitude of cloud base, $\rho(z)$ is the air density, c_p the specific heat of air at constant pressure, \mathcal{L} the latent heat of vaporization of water, $\Gamma_s(z)$ the pseudoadiabatic lapse rate, and Γ the dry adiabatic lapse rate; M^* may be regarded as a kind of theoretical upper limit of the water content. Although the water contained in any particular cloud may deviate considerably from M^* , (28) nevertheless provides an objective method of prescribing M . Sheppard found that M^* tended to be somewhat larger than the accepted values of cloud water content, so in his calculations he used $M(h) = M^*(h)/2$. The columnar liquid water L

TABLE 2. Retrieval coefficients for Method 2 [Eq. (16)].

	τ_{O1}	K_{V1} (cm^{-1})	K_{l1} (cm^{-1})	τ_{O2}	K_{V2} (cm^{-1})	K_{l2} (cm^{-1})
Mean	0.01214	0.03748	0.81402	0.02444	0.01283	1.82173
Standard deviation	0.00081	0.00064	0.31083	0.00164	0.00159	0.68760

TABLE 3. Retrieval coefficients for Methods 2, 3, and 5. (All entries in cm.)

	a_0	a_1	a_2	b_0	b_1	b_2
Method 2	-0.01315	-0.22186	0.64807	-0.03855	31.4995	-14.0752
Method 3	-0.01947	0.22198	0.35599	0.00368	31.6366	-14.3426
Method 5	-0.01181	-0.16566	0.53743	-0.03770	31.2563	-13.8028
Method 3 (archival data)	-0.01828	-0.04392	0.53979	0.04312	28.203	-12.217 (Jan)
	-0.01725	-0.12585	0.60192	0.02693	28.993	-12.624 (Feb)
	-0.01755	-0.15419	0.63117	0.02223	29.219	-12.564 (Mar)

in the cloud is then obtained by integrating $M(h)$ over the vertical extent of the cloud. It should be mentioned that there are alternatives to this way of prescribing the cloud water (e.g., Decker et al. 1978) but that all methods are somewhat arbitrary. We have followed Sheppard and used $M = M^*/2$ in determining retrieval coefficients.

d. Soundings used in calculating the coefficients

Sheppard calculated the coefficients in (18) for Method 3 from the same archival set of radiosonde data that he used in determining the empirical relation (27) for the mean radiating temperature. We calculated the retrieval coefficients for this method and all the others using the set of special soundings taken at the radiometer site during the CASP project. These soundings were taken to enable direct comparisons of the temperature profiles and columnar vapor measured by radiosonde and by radiometer. We used 108 of these in the comparisons, as reported earlier (Blaskovic et al. 1989). The same soundings were used in determining the new retrieval coefficients. It is worth noting that the soundings we used were taken at irregular times, more often than not during conditions of disturbed weather, whereas the archival data that Sheppard used were regular, twice-daily soundings.

e. Summary of the coefficients

Table 1 gives the mean values and the standard deviations of the retrieval coefficients in (14) for Method 1, based on the set of 108 CASP soundings. Although the optical thickness of oxygen at a frequency of 31.65 GHz is seen to average about twice that of water vapor, the variability about the mean is much less for oxygen than for the vapor. The mean absorption coefficient of the liquid also has a high degree of variability. It may be noted that (14) is of the form $L = A_0 + A_2\tau_2$, where $A_2 = 1/K_l$ and $A_0 = -(\tau_V + \tau_O)/K_l$. From the entries in Table 1, $A_0 = -0.02112$ cm and $A_2 = 0.54893$ cm.

TABLE 4. Retrieval coefficients for Method 4.

b_0 (cm)	b_1 (cm)	b_3
-0.34089	27.0015	-25.6828

Table 2 lists the coefficients in (16) for the two-channel physical method. The variability of the liquid coefficients is seen to be more significant than that of the vapor or of the oxygen.

Table 3 gives the retrieval coefficients in (18) for Method 3. It includes for comparison the coefficients for Methods 2 and 5, which can be written in the form of (18). For Method 2 these coefficients are given by (17); for Method 5 they follow from (25) and (26). Table 3 also includes the retrieval coefficients calculated by Sheppard from the Shelburne archival radiosonde records. He calculated separate coefficients for each of the three months, as indicated. These are the coefficients that were used in the earlier reports on the CASP radiometer data (Blaskovic et al. 1989; Wei and Rogers 1988).

Table 4 gives the vapor retrieval coefficients for Method 4, as they appear in (22). The three empirical regressions (denoted by A, B, and C in the text) on which this relation rests are explained in the Appendix. The fourth empirical regression (denoted D) given by (23), on which Method 5 depends, is also explained

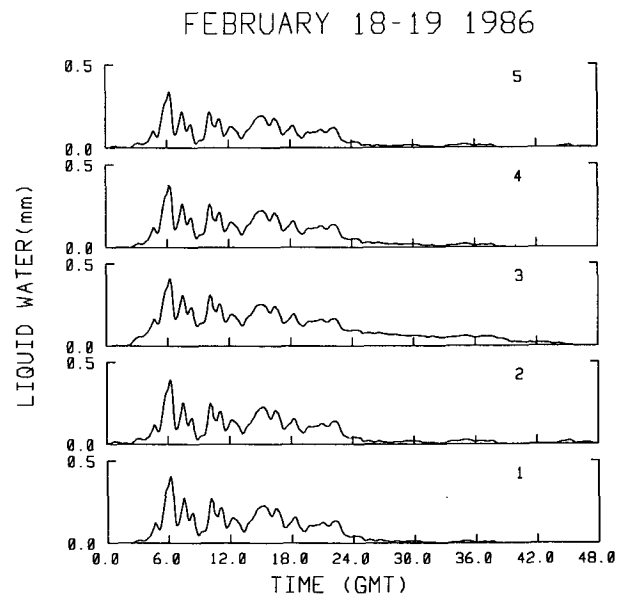


FIG. 1. Record of vertically integrated cloud liquid over a 48-hour period. Curves 1, 2, 3, 4, 5 correspond to the five retrieval methods explained in the text.

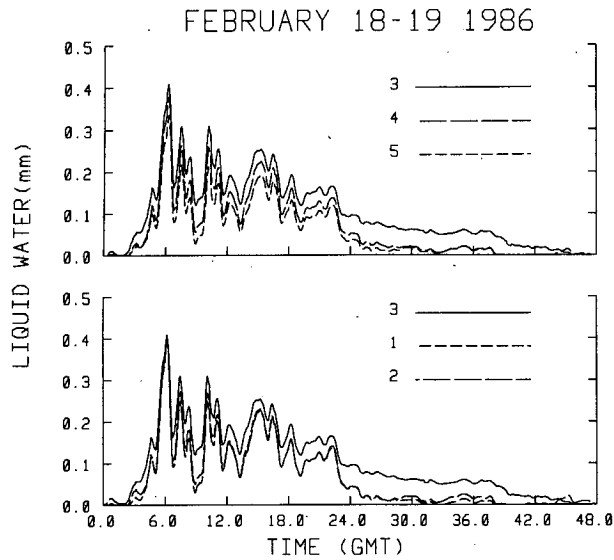


FIG. 2. Same data as in Fig. 1, except plotted to enable comparison.

in the Appendix. The proportionality factor r , introduced in Method 4, was determined by comparing τ_1 at 20.6 and 31.65 GHz for all soundings using the cloud model. The average value of r was found to be 0.4416 with a standard deviation of 0.0454. This average was used in applying Methods 4 and 5.

6. Comparison of the five methods

As an example, Fig. 1 shows the records of vertically integrated cloud liquid for a 48-h period of CASP observations, using the different methods. Conditions were overcast for essentially all of this period, but there was no precipitation at the radiometer site. The radiometer observations are at a frequency of 11 times per hour. A 6-point running average, corresponding to an averaging time of approximately a half an hour, was used to produce the curves in Fig. 1. These curves indicate a high degree of overall agreement in the different methods. Figure 2, in two parts, allows a more direct comparison. It shows that for this particular example Method 3 tends to give rather higher estimates of L than the other methods, especially during the times when L is small. All methods are equally effective in indicating the general trends in cloud liquid, but the precise amount of liquid varies from one method to the other. In the absence of independent information on the liquid amount, there is no way of judging which method is most accurate.

With noisy or otherwise questionable data excluded, there were a total of 12 403 individual radiometer observations during the 52-day period from 21 January to 13 March 1986. At the sampling rate of 11 h^{-1} , this number of observations corresponds to 1128 h, or 47 days out of the total 52. This period included fair and

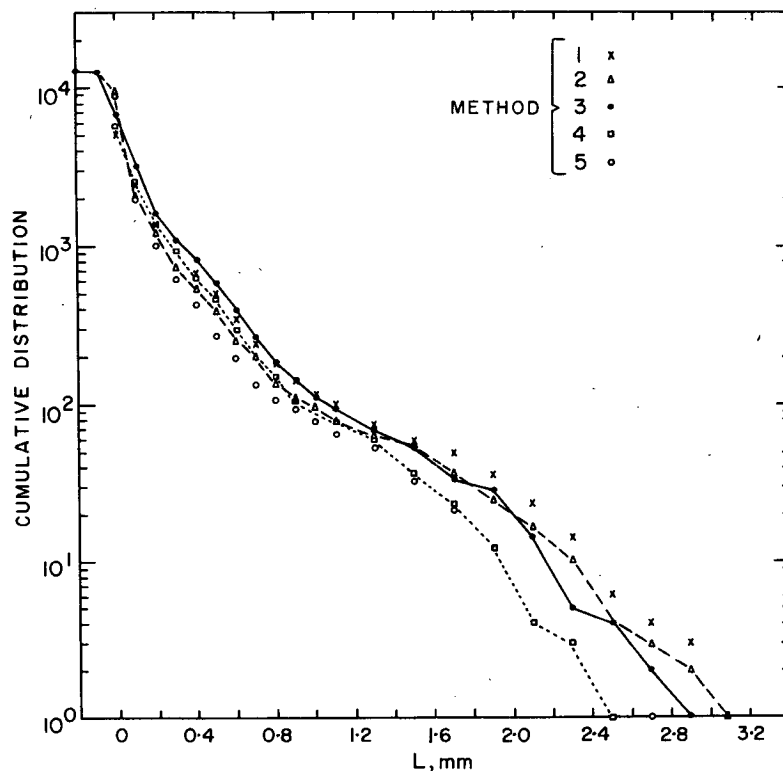


FIG. 3. Cumulative distributions of integrated liquid by the five methods. To avoid confusion, not all the points are connected by lines.

TABLE 5. Error estimates in retrieved columnar liquid water for each of the five methods.

Class*	N	L (cm)	Mean deviation (cm)					Rms deviation (cm)				
			1	2	3	4	5	1	2	3	4	5
I	280	0.0173	1.1×10^{-3}	3.0×10^{-3}	2.1×10^{-3}	3.6×10^{-4}	-3.1×10^{-4}	5.7×10^{-3}	5.1×10^{-3}	5.2×10^{-3}	3.4×10^{-3}	3.7×10^{-3}
II	45	0.196	2.2×10^{-2}	2.0×10^{-2}	-6.4×10^{-3}	-1.8×10^{-2}	-1.4×10^{-2}	3.0×10^{-2}	2.9×10^{-2}	1.8×10^{-2}	2.6×10^{-2}	2.4×10^{-2}
III	21	0.387	4.8×10^{-2}	4.4×10^{-2}	-1.7×10^{-2}	-1.9×10^{-2}	-2.3×10^{-2}	5.9×10^{-2}	5.7×10^{-2}	3.1×10^{-2}	3.6×10^{-2}	3.6×10^{-2}
IV	12	0.732	1.2×10^{-1}	1.1×10^{-1}	-2.3×10^{-2}	-2.3×10^{-2}	-1.8×10^{-2}	1.4×10^{-1}	1.4×10^{-1}	4.9×10^{-2}	3.3×10^{-2}	5.3×10^{-2}
V	358	0.0428	1.0×10^{-2}	1.1×10^{-2}	-9.5×10^{-4}	-3.9×10^{-3}	-3.9×10^{-3}	3.1×10^{-2}	3.1×10^{-2}	1.4×10^{-2}	1.4×10^{-2}	1.6×10^{-2}

* I: $0 \leq L < 0.1$, II: $0.1 \leq L < 0.3$, III: $0.3 \leq L < 0.5$, IV: $0.5 \leq L < 1.0$, V: $L < 1.0$.

foul weather, and 14 of the CASP Intensive Observing Periods (IOP 2-15). Cloud liquid was computed for every one of these observations using the five different methods. Histograms of the results are plotted in Fig. 3. Although they are not physically meaningful, this figure includes for completeness the negative values of L that the calculations occasionally produce. Overall, the largest values of L are given by Method 1 and the smallest by Methods 4 and 5. For $L \leq 1.5$ mm, the distributions are rather close to one another, the variation of L for a given frequency of occurrence amounting to only ± 0.1 mm. The deviations are greater for larger values of L , but this may only be a reflection of the small sample size.

7. Assessment of accuracy

Because independent measurements of cloud liquid are not available, we turn to simulation as a way of evaluating the accuracy of the different radiometric methods of inferring liquid amount. Given the vertical distributions of pressure, temperature, humidity, and cloud water content, as specified by a sounding and a cloud model, we are able to solve for $\tau(0, \infty)$ and T_b for a particular radio frequency using (3) and (5) and knowledge of the absorption coefficients of the oxygen, the water vapor, and the cloud. Using the values of τ so determined, we can then go to the different retrieval equations and solve for V and L . [For Method 4, L in the retrieval equation (22) is determined from the regression on T_b .] We can also determine the "true" values of V and L by direct integration of the sounding. Thus, for any sounding, we can compare the "true" values of V and L with the values inferred using the different retrieval methods.

To assess the accuracy of the methods we used the archival, twice-daily soundings from Shelburne, Nova Scotia (about 150 km from Shearwater) during the months January, February, and March, from the period 1 January 1985-11 January 1987. We eliminated the soundings in which there were obvious errors in the data or in which the vertically integrated adiabatic liquid water content, L^* , exceeded 1 cm. Of the remaining 280 soundings, 127 were such that the relative humidity never exceeded 95% and hence cloud was not present. The adiabatic cloud liquid L^* was determined for the rest from the integral over height of $M^*(h)$. For each of these soundings, from one to four different values of the integrated cloud liquid, L , were then prescribed, each a specified fraction of L^* , and each providing a different simulation. No single sounding was used to generate more than one value of L in each of the ranges: $0 \leq L < 0.1$; $0.1 \leq L < 0.3$; $0.3 \leq L < 0.5$; and $0.5 \leq L < 1.0$ cm. The values of L ranged from $L^*/10$ to L^* , generating in total 358 simulations from the 280 soundings. In this way, the sample was made larger than it would have been for only one value of L per sounding.

TABLE 6. Error estimates in retrieved columnar vapor. Class ranges for L as in Table 5 for each of the five methods.

Class	N	\bar{V} (cm)	Mean deviation (cm)				Rms deviation (cm)			
			2	3	4	5	2	3	4	5
I	280	0.83	-4.8×10^{-3}	2.6×10^{-2}	-1.6×10^{-2}	1.0×10^{-3}	2.4×10^{-2}	3.8×10^{-2}	1.2×10^{-1}	2.4×10^{-2}
II	45	1.51	-5.0×10^{-2}	-9.1×10^{-2}	1.3×10^{-1}	1.1×10^{-2}	8.3×10^{-2}	1.2×10^{-1}	2.6×10^{-1}	6.8×10^{-2}
III	21	1.75	-1.3×10^{-1}	-2.6×10^{-1}	-1.0×10^{-1}	-9.9×10^{-3}	1.5×10^{-1}	2.7×10^{-1}	3.7×10^{-1}	8.8×10^{-2}
IV	12	1.83	-6.0×10^{-1}	-8.8×10^{-1}	-2.2×10^{-1}	-3.5×10^{-1}	6.7×10^{-1}	9.5×10^{-1}	1.2	4.2×10^{-1}
V	358	1.00	-3.8×10^{-2}	-3.5×10^{-2}	-1.0×10^{-2}	-1.0×10^{-2}	1.3×10^{-1}	1.9×10^{-1}	2.7×10^{-1}	8.7×10^{-2}

Results of the simulations are summarized in Table 5. The total sample of 358 was separated into the four ranges of liquid amount specified above. For each range and each method, the table includes the mean difference between the retrieved value and the "true" value of L , and the rms difference. Also given (as Class V) are the overall values of these two quantities. The first liquid category, it should be noted, includes the fairly numerous cloud-free samples, and strongly influences the overall statistics.

The table shows that the mean deviations, which indicate bias in the estimates, have their largest values for the two physical methods (Methods 1 and 2) for the highest category of L . Overall, the physical methods show a positive bias while the three statistical methods have smaller but negative biases. Method 3 overall has the smallest bias, but for all statistical methods the biases amount to less than 10 percent of the mean liquid amount, 0.428 mm. The rms deviations are also larger for the two physical methods, exceeding by about a factor of 2 the rms deviations for the statistical methods. These deviations are much the same for all the statistical methods, amounting overall to slightly more than 30 percent of the mean value of L . By category of liquid, it is seen that the ratio of rms deviation to mean liquid is less for large water contents than for the smaller ones. From this comparison, we conclude that the three statistical methods as a group are more accurate than the physical methods, but that none of these three has a clear advantage over the others.

Table 6 summarizes the ability of each of the methods to estimate the columnar vapor. As before, the cases are stratified by liquid amount, and both the mean and rms deviations are given. (Method 1 has no entries, because vapor is not estimated in the single-frequency method.) For all methods, the biases and the rms deviations are seen to increase with increasing liquid amount. All methods underestimate the actual amount of vapor (biases are negative, overall), but the biases are a small fraction of the mean. The rms errors are least for Method 5 and are the largest for Methods 3 and 4.

8. Conclusions

The results give confidence in radiometric estimation of columnar cloud liquid. The three statistical methods

are more accurate than the physical methods for all but the smallest values of liquid amount. For $L \geq 0.3$ mm, the statistical methods gave small negative biases, amounting always to less than 5 percent of the mean value of L in each of the classes of liquid amount. Also for $L \geq 0.3$ mm, the rms errors of the statistical methods are less than 10 percent of the mean value of L . For comparison with the findings of Snider et al. (1980), cited earlier, we may note that the overall bias using the method he employed, our Method 3, when all the values of L in the simulations are included, is -2.2 percent, and the overall rms error is 33 percent. The corresponding figures from Snider et al. are $+11$ percent for the bias and 50 percent for the rms error as a fraction of the overall mean value of L . While the magnitudes are comparable, no further discussion seems warranted because of the different approaches in the two studies. The previous work was an experimental test; our study was a simulation.

Although the three statistical methods we considered gave similar accuracy in determining the integrated liquid, Method 5 emerged as slightly superior for determining the integrated vapor. Our results are based on simulations for one location and the winter season. While it is not clear that Method 5 would be the optimum in all situations, our findings support the conclusion that statistical retrievals of columnar liquid water and water vapor are generally more accurate than physical retrievals.

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APPENDIX

Empirical Regressions for Methods 4 and 5

Each of the empirical regressions is based on the set of 108 CASP soundings with the cloud model as explained in the text, using for the liquid water content $M = M^*/2$.

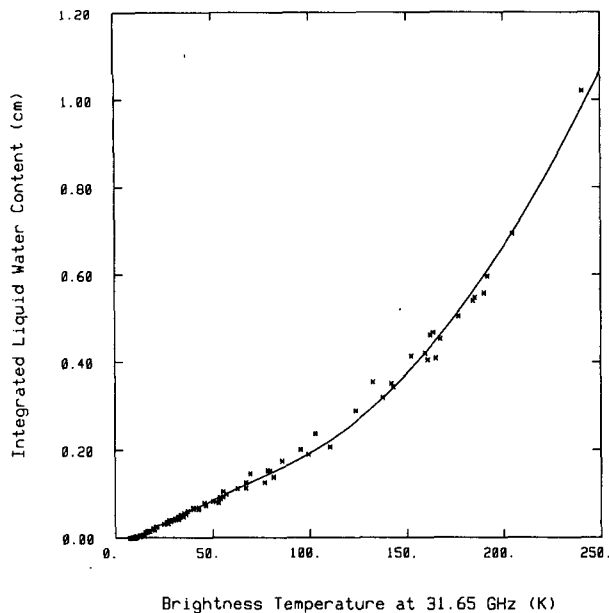


FIG. A1. Relationship between integrated cloud liquid and brightness temperature at 31.65 GHz, for regression A.

Figure A1 is a plot of L versus T_b at 31.65 GHz. The relation is linear to a good approximation for small values of L , and can be approximated by a second-order polynomial for the larger values. The curve shown is the relation we used for regression A [Eq. (19)], which is linear to $T = 90$ K and quadratic there-

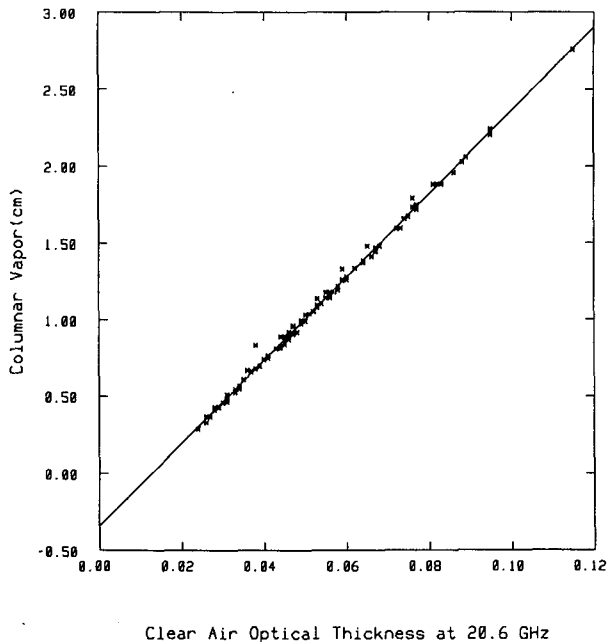


FIG. A3. Relationship between integrated water vapor and optical thickness at 20.6 GHz, for regression C.

after. The coefficients for this regression are listed in Table A1.

Figures A2, A3, and A4 indicate the linear regressions B, C, and D. The corresponding regression coefficients are given in the table.

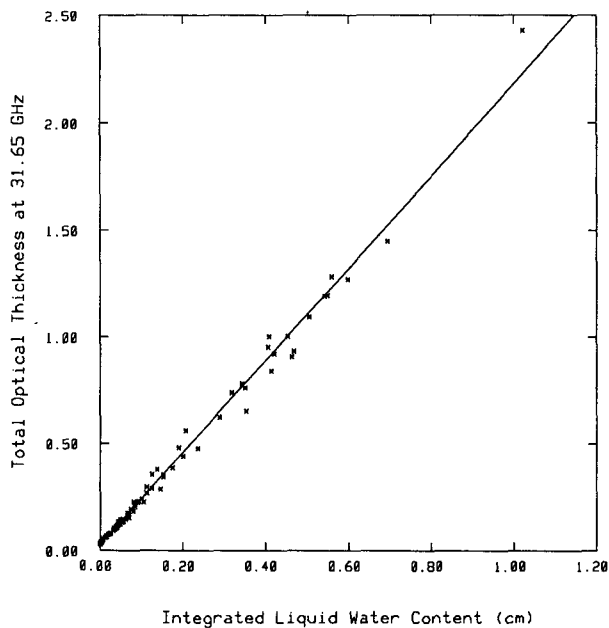


FIG. A2. Relationship between integrated cloud liquid and optical thickness at 31.65 GHz, for regression B.

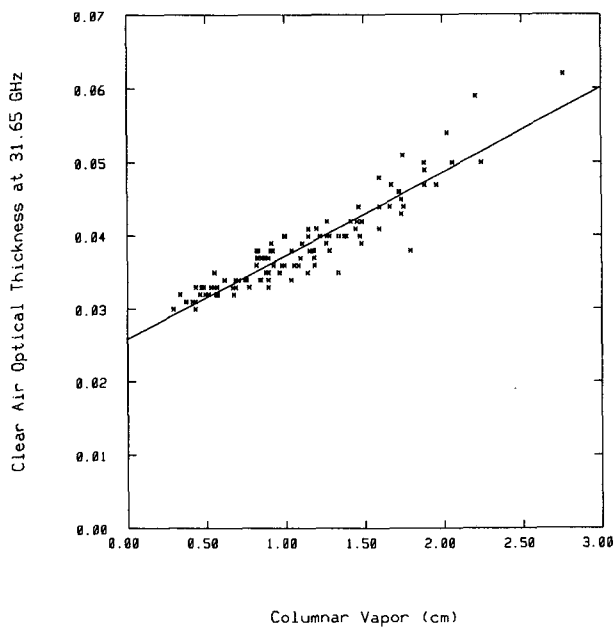


FIG. A4. Relationship between integrated water vapor and optical thickness at 31.65 GHz, for regression D.

TABLE A1. Regression coefficients for Methods 4 and 5.

Regression equation	Symbol	Regression coefficients		Rms deviation		Correlation coefficient	
		$T_{bz} \leq 90$ K	$T_{bz} > 90$ K	$T_{bz} \leq 90$ K	$T_{bz} > 90$ K	$T_{bz} \leq 90$ K	$T_{bz} > 90$ K
$L = a + bT_{bz} + c(T_{bz})^2$ (in cm)	a	-0.01943	0.1598				
	b	0.002087	-0.001891	0.00495	0.0256	.994	
	c	0	0.000022				
$\tau_2 = p + qL$	p		0.03015				
	q		2.1539	0.0365			.996
$V = m + n\tau_{a1}$ (in cm)	m		-0.3409				
	n		27.0015	0.0264			.999
$\tau_{a2} = x + yV$	x		0.02586				
	y		0.01142	0.00223			.930

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