Resolution and accuracy of a multi-frequency scanning radiometer for temperature profiling

ED R. WESTWATER¹, YONG HAN¹, and FRED SOLHEIM²

¹CIRES, University of Colorado/NOAA Environmental Technology Laboratory, Boulder, CO 80303, USA.
²Radiometrics Corporation, Boulder, CO 80303, USA.

Abstract - A new multi-frequency scanning radiometer has been constructed by the Radiometrics Corporation. The accuracy of the system is evaluated for a climatology whose temperature profiles are difficult to recover by radiometric retrievals - Barrow, Alaska, USA. It is found that rms retrieval accuracies of less than 1 K are possible up to about 3 km. In addition, the vertical resolution of the system is evaluated using two variations of the Backus-Gilbert technique. The first uses only information from the radiometric weighting functions, while the second also includes a priori statistical information on temperature profiles.

1. INTRODUCTION

In experiments conducted on a FLoating Instrument Platform [1,2], at two Water Vapor Intensive Operating Periods at the Atmospheric Radiation Measurement (ARM) Program's Southern Great Plains Central Facility, and at the Boulder Atmospheric Observatory [3], an angular-scanning single-frequency radiometer accurately measured low altitude temperature profiles. ARM has purchased from the ATTEX Corporation, Moscow, Russia, and is currently operating a similar instrument on the North Slope of Alaska/Adjacent Arctic Ocean (NSAAO) Cloud And Radiation Testbed (CART) site. This instrument derives temperature profiles up to about 300 m with a rms accuracy of about 1.0 K. To extend the accuracy of such instruments to higher altitudes, Radiometrics Corporation, Boulder, Colorado, USA, is developing a multi-frequency scanning radiometer that operates in the frequency region of the 60 GHz oxygen absorption band. This paper describes the instrument as well as a theoretical evaluation of the technique.

2. DESCRIPTION OF INSTRUMENT

The Radiometrics ground-based TP/WVP-3000 portable water vapor and temperature profiling radiometer measures well-calibrated brightness temperatures from which are derived profiles of temperature, water vapor, and limited resolution profiles of cloud liquid water from the surface to 10 km. Descriptions of the system are given in [4,5] and only a short summary of instrument characteristics is given here. Novel characteristics of the system include a very stable local oscillator, an economical way to generate multiple frequencies, and the multi-frequency
and scanning capability. The radiometer system consists of two separate subsystems in the same cabinet that share the same antenna and antenna pointing system. A highly stable synthesizer acts as receiver local oscillator and allows tuning to a large number of frequencies within the receiver bandwidth. The water vapor profiling (WVP) subsystem receives thermal emission at five selected frequencies between 22 and 30 GHz. The temperature profiling (TP) subsystem uses sky observations at seven selected frequencies between 51 and 59 GHz. Surface meteorological sensors measure air temperature, barometric pressure, and relative humidity. To improve measurement of water vapor and cloud liquid water density profiles, cloud base altitude information is obtained with an infrared thermometer. In this paper, we only evaluate the temperature sensing characteristics of the instrument. The salient characteristics of the temperature channels are shown in Table 1.

| Table 1. Characteristics of Radiometrics TP-3000 Angular-Scanning Radiometer |
|-------------------------|-----------------------------|
| **Frequencies (GHz) for water vapor and liquid sensing** | 22.235, 23.035, 23.835, 26.235, 30.00 |
| **Frequencies (GHz) for Temperature Sensing** | 51.25, 52.85, 53.85, 54.94, 56.60, 57.29, 58.80 |
| **Absolute Accuracy (K)** | 0.5 |
| **Sensitivity (K)** | 0.25 |
| **FWHP beamwidth (deg)** | 2.2 - 2.4 |
| **Gain (dB)** | 36 - 37 |
| **Sidelobes (dB)** | < -26 |

3. TEMPERATURE WEIGHTING FUNCTIONS

For the frequencies considered here, the main radiators in the atmosphere are water vapor (H₂O), oxygen (O₂), and cloud liquid droplets. Because the O₂ concentration in the troposphere is relatively stable and therefore, can be inferred from air pressure and temperature, the brightness temperature \( T_b \) is a function of the vertical profiles of air pressure \( p \), temperature \( T \), water vapor density \( \rho_v \), and cloud water content \( \rho_L \) at specified frequency \( v \) and elevation angle \( \theta \), i.e., \( T_b = T_{b,v,\theta}(p, T, \rho_v, \rho_L) \). The sensitivity of the brightness temperature \( T_b \) to any of the four parameters, represented by \( x \), may be evaluated by the so-called weighting function \( W_x(z) \), which is the response (change) of the brightness temperature to a unit positive perturbation of the profile in a 1-km-thick layer at a height \( z \). The temperature weighting function \( W_T(z) \) is given in [6]

\[
W_T(z) = \frac{1}{\sin(\Theta)} \alpha(z) e^{-\tau(0,z)/\sin(\Theta)}
+ \frac{1}{\sin(\Theta)} e^{-\tau(0,z)} \frac{\partial \alpha(z)}{\partial T} [T(z) - T_{bb} e^{-\tau(0,\infty)/\sin(\Theta)} - \int_{z}^{\infty} T \alpha e^{-\tau(z,z')/\sin(\Theta)} dz']
\]
In (1), \( \alpha \) is the absorption coefficient, \( \tau(z, z') \) is the optical depth between the heights \( z \) and \( z' \), \( T \) is the temperature, \( T_{eb} \) is the cosmic big bang brightness temperature (K), and \( \theta \) is the elevation angle. The first term of the above equation is due to the temperature increase of the radiation source function in the layer at \( z \), and the remaining three terms are due to the change of the absorption coefficient caused by the increase of the layer temperature. The first of the three terms is the increase (decrease) of radiation in the layer at \( z \) due to the increase (decrease) of the absorption coefficient; the other two terms count for the decrease (increase) of radiation, coming from higher layers and the space, caused by the increase (decrease) of the absorption at layer \( z \). Examples of temperature weighting functions as a function of elevation angle are shown in Figure 1 for the most transparent and most opaque channel of Table 1. All temperature weighting functions for ground-based tropospheric remote sensing decrease rapidly with height, but with different rates and strengths. It is the differentiation among the decreasing rates that makes possible temperature profile measurements with good vertical resolution below about 300 m. With the exception of the most transparent channels, the response to water vapor and clouds is negligible. Numerical evaluation of these weighting functions, as well as accuracy and resolution analysis that is presented later, is based on the millimeter wave propagation model of Liebe [7].

4. SINGULAR VALUE DECOMPOSITION OF WEIGHTING FUNCTIONS

As might be expected from the Figure 1, there is a high degree of redundancy between the various frequency and angular weighting functions. To characterize this redundancy, and also to compress the amount of useful information in the measurements, we performed a singular value decomposition (SVD) [8] of the weighting functions. We reduced the weighting functions to a matrix \( A \) based on an equally spaced quadrature of \( m = 301 \) points between 0 and 3 km. For the SVD, we write

\[
A = U \Lambda V^T
\]  

(1)
where \( A \) is an \( n \times m \) matrix, \( U \) is an \( n \times m \) matrix whose columns contain the left singular vectors of \( A \), \( \Lambda \) is an \( m \times m \) diagonal matrix whose elements are the singular values of \( A \), and \( V \) is and \( m \times m \) matrix whose columns contain the right singular vectors of \( A \). In an ill posed problem, in which many of the singular values of \( \Lambda \) are below the noise level, it is desirable to use only the vectors whose singular values are significant. For a selection of 50 angles and 7 frequencies; i.e., 350 measurements, it was determined that all of the information could be compressed to an accuracy of 1 part in \( 10^6 \) by 9 singular functions. The number of singular functions that are used depends on the minimum value below which the singular values are considered insignificant. This minimum value may be determined by the computational precision, as we did here.

5. ACCURACY ANALYSIS

The accuracy of a ground-based system is a function of the instrument characteristics, as well as the climatological regime in which the instrument operates. The primary instrumental characteristics are the absolute accuracy in measuring the brightness temperature and the beamwidth. The primary climatological factors include the strengths, heights, and frequency of occurrence of inversions, especially elevated inversions. We have performed an error analysis of the multi-frequency system for four diverse climatologies of the USA: Barrow, Alaska; Fairbanks, Alaska; the CART site in central Oklahoma, and Denver, Colorado. For brevity, we will only show results for the location with the highest variability: Barrow, Alaska. Our simulations showed that the accuracies could vary by almost a factor of two between the various climatologies.

About 5000 radiosondes over about 5 year period from each of the four sites are collected as statistical ensembles. Calculations of brightness temperatures at the frequencies shown in Table 1, as well as 50 elevation angles evenly distributed between 5 and 90 degrees were carried out. The lower limit of 5 degrees was chosen to be about 2 beamwidths, so that ground effects were small. Gaussian noise was added to the calculated brightness temperatures with the assumption that there are no correlations between the measurements. Surface air temperature \( T_s \) measurements were also included in the measurement vector. Figure 2 shows the expected accuracy of deriving temperature profiles from the seven channel radiometer, for an assumed noise levels of 0.1 and 0.5 K. For comparison, the accuracy derived from predicting the temperature by \( T_s \) is also shown.

6. RESOLUTION ANALYSIS

The characterization of vertical resolution in radiometric atmospheric profiling has been rather difficult to define. Definitions of resolution, such as the Rayleigh criterion that is applied in optics, or the width of a radar pulse volume in radar meteorology, are not readily applicable to the overlapping weighting functions of radiometry. One definition, as was applied by Backus and Gilbert [9] to remote sounding of the solid earth, is applicable, and, as extended by Rodgers
[10] to include a priori statistics, lends itself rather well to radiometry. The basic idea is, for each height at which a retrieval is desired, to construct a linear combination of weighting functions, that approximates as closely as possible, some function that has ideal resolution characteristics; e.g., a Dirac delta function, or a gaussian with small standard deviation centered about the point in question. We follow Backus and Gilbert and chose a function called the spread as a measure of resolution. A tradeoff between spread and error is a key element of their theory.

### 6.1 Minimum spread of the seven channel scanning radiometer

We evaluated minimum values of the spread for 11 altitudes between 0 and 3 km. First, we projected the weighting functions for 7 frequencies and 50 angles onto the first nine singular functions, and evaluated the spread for these projections. The results are shown in Table 2. We note that the resolution, without the input of a priori data, is roughly equal to the height in question.

<table>
<thead>
<tr>
<th>height (km)</th>
<th>Spread (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>0.020</td>
</tr>
<tr>
<td>0.050</td>
<td>0.046</td>
</tr>
<tr>
<td>0.100</td>
<td>0.097</td>
</tr>
<tr>
<td>0.150</td>
<td>0.151</td>
</tr>
<tr>
<td>0.250</td>
<td>0.232</td>
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<tr>
<td>0.500</td>
<td>0.469</td>
</tr>
<tr>
<td>0.750</td>
<td>0.789</td>
</tr>
<tr>
<td>1.000</td>
<td>0.953</td>
</tr>
<tr>
<td>1.500</td>
<td>1.651</td>
</tr>
<tr>
<td>2.000</td>
<td>1.770</td>
</tr>
<tr>
<td>2.500</td>
<td>3.544</td>
</tr>
</tbody>
</table>

### 6.2 Backus-Gilbert method using a priori statistics

As discussed in [10], information other than radiation measurements can be incorporated into the spread-error analysis. For example, we frequently have a priori climatological statistics, a forecast from a numerical model, or some other source of information. To use such information, the error characteristics of the source of information are needed. Here, we have introduced the covariance matrix of temperature fluctuations, conditioned on the surface temperature measurement. We evaluated the spread vs. error of the measurement system.
consisting of the seven-channel scanning radiometer (assumed noise level of 0.5 K) and the climatological mean, conditioned on $T_s$, determined from the a priori statistics of Barrow, Alaska. Again, the 350 weighting functions were projected onto the first nine singular functions. The tradeoff curves are shown in Figure 3. We note that, as expected from the weighting functions shown in Figure 1, the vertical resolution (spread) becomes poorer with height, but that good resolution and accuracy are achievable to about 300 m. As another figure of merit, we show in Figure 4 the spread at which the error standard deviation becomes 1 K rms.

![Figure 3](image3.png) ![Figure 4](image4.png)

**Figure 3.** Spread vs. temperature error standard deviation as for several heights. The height labels from 0 to 10 indicate the heights: 0.025, 0.05, 0.1, 0.15, 0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5 km, and the assumed radiometric resolution is 0.5 K rms.

**Figure 4.** Spread vs. height for a temperature error standard deviation of 1.0 K rms.

### 7. CONCLUDING REMARKS

We evaluated the expected temperature profile retrieval accuracy during clear conditions, and for a severe arctic environment, of a scanning seven-frequency microwave radiometer, that has been developed for ARM by the Radiometrics Corporation. The results show that rms retrieval accuracies of better than 1 K rms are achievable up to 3 km with this system, although the vertical resolution degrades rapidly above 500 m. Although the results were obtained under the assumptions of clear conditions, the complete TP/WVP-3000 radiometer has water vapor and cloud channels, that should allow roughly the same temperature-profiling accuracy to be obtained during non-precipitating conditions. Previous experience obtained with a 6-channel zenith-viewing Radiometric Profiler showed that the effects of moisture could be taken into account with a dual-frequency water vapor radiometer [6]. The multi-frequency WVP should only improve on this accuracy.

Finally, we note that the Radiometric Corporation's TP/WVP-3000 was used at the NSA/AAO in Barrow, Alaska, during March 1999, along with two single-frequency scanning $O_2$-band radiometers. The data taken during this experiment should allow an excellent evaluation of the respective instruments during extreme arctic conditions.
REFERENCES


5. F. Solheim, J. Godwin, and R. Ware, Passive ground-based remote sensing of atmospheric temperature, water vapor, and cloud liquid profiles by a frequency synthesized microwave radiometer, Meteorologische Zeitschrift, 7, 370-376, (1998).


